Phreatic Karst Conduit Flood Pulse Modeling

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Introduction

Karst aquifers supply about 25% of the world's population with water [1], including nearly 90% of Florida's drinking water [2]. Input of conduit water to the matrix throws the water in the matrix out of chemical equilibrium fostering a variety of geochemical interactions, including speleogenesis [3, 4]. Interaction between conduit and matrix water also has implications for karst aquifer biology, residence time distributions, and contaminant transport [5]. A conduit flood is one mechanism that pushes conduit water into the matrix. Conceptually we borrow from Cooper and Rorabough's [6] seminal model of bank storage of fluvial flood water to model a flood wave passing through a phreatic karst conduit. This event is is modeled in the matrix using a two dimensional time dependent model of Darcy's law.

Background on Conceptual Basis

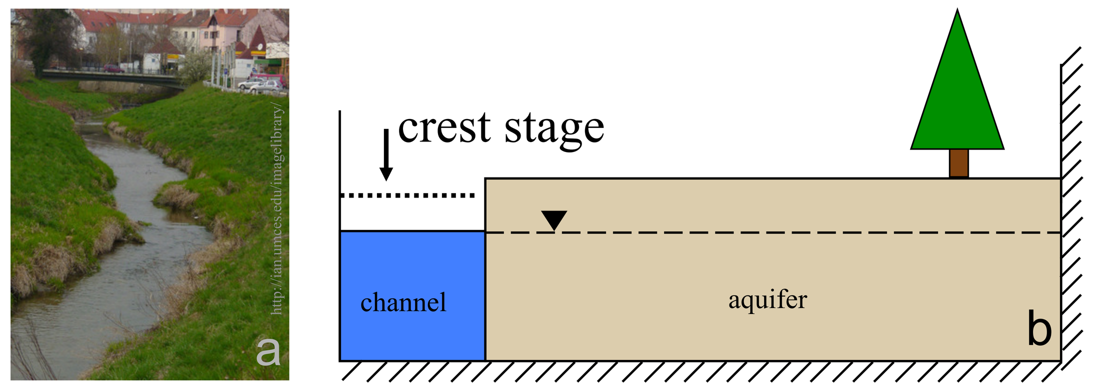

Figure 1: Cooper and Rorabaugh (1963) geometry, a) example fluvial system; b) 1-dimensional model geometry.

Over forty years ago (1963) Cooper and Rorabaugh (CR) [6] of the US Geological Survey developed a simple but important one-dimensional, linear model of streambank storage of a passing flood wave in a surface stream or channel. Given the flood wave history, the model explained water level changes in the permeable aquifer bounding the channel, estimated flux rates between channel and aquifer, and modeled the time history of the volume of bank storage. Our karst conduit flood model conceptually follows the CR model. We employ their "symmetric" flood wave model,

$$h_c(t) = \begin{cases} \frac{H}{2}\left[1-\cos\left(\frac{2\pi t}{\tau}\right)\right] & t \le \tau \\ 0 & t > \tau \end{cases} \qquad (1)$$

to force the system, where: $h_c(t)$ is the time history of head in the conduit; H is the head at the peak of the flood wave; τ is the duration of the flood wave (Figure 1c). CR also considered non-symmetric flood waves, which we will leave to future work for the karst conduit model. The symmetric model is by itself very revealing of behavior [7].

Karst Conduit Model

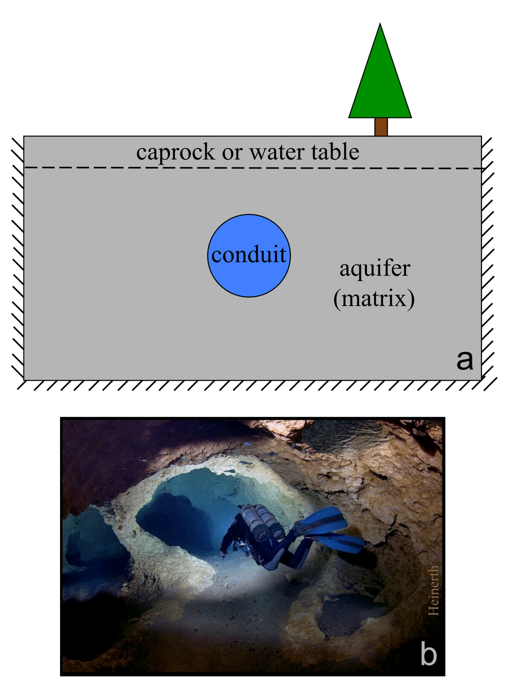

Figure 2: Karst Conduit Model Geometry, a) 2-dimensional model geometry; b) example karst conduit.

Since karst conduits are analogous to surface streams [8], we borrowed from the conceptual approach in the CR model to study matrix-storage in a phreatic (fully saturated) cave conduit. In our approach we use CR's "symmetric" flood wave (Equation 1) but a different geometry; one with a long, circular conduit located somewhere below the top boundary of a karst aquifer, Figure 2a, b. Consistent with CR's conceptual approach, we assume that temporal changes perpendicular to the conduit are more important than those parallel [9], permitting a two-dimensional cross-sectional model.

About the model:

- 2-dimensional cross-sectional model.
- The domain is a water saturated cross-section of the conduit matrix system, taken normal to the conduit axis, x.
- Top boundary: caprock or water table.
- Dependent variables: hydraulic head, solute-tracer concentration (conservative).
- Independent variables: time, location.
- Conduit behavior is prescribed and not modeled.
- Flow in matrix is modeled using Darcy's Law. The matrix, consisting of a porous rock, fractures, and fissures, is treated as a single porosity, isotropic, homogeneous porous medium.

COMSOL Modeling

In this poster we present only numerical model results, achieved using the finite element method through the commercial code COMSOL Multiphysics. One advantage of using a computer model, rather than an analytical solution, is the ease with which we can model sequential floods. Using a numerical model also allows us to include sensitivity to recharge or leakage across the top boundary by making the top boundary a prescribed flux boundary.

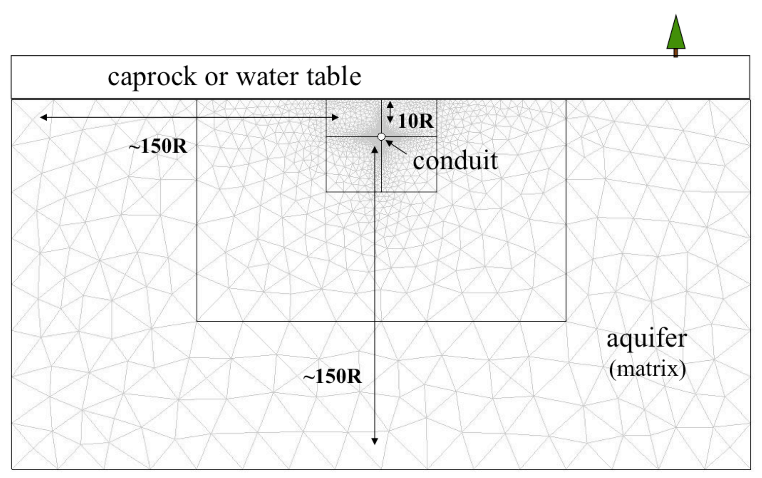

Figure 3: COMSOL model domain and finite element mesh.

Flux of Flood Water

The initial condition of the matrix is assumed to be in local equilibrium with the conduit (there is head gradient in the unmodeled x-direction). A flood wave, measured by an increase of head in a conduit, passes through the conduit. The result is a high flux of conduit water to the matrix for a duration slightly shorter than the flood pulse. As the flood recedes, the head in the conduit begins to drop, and the hydraulic gradient near the conduit reverses. The conduit drains a low flux of matrix water over a time period much longer than the flood pulse duration.

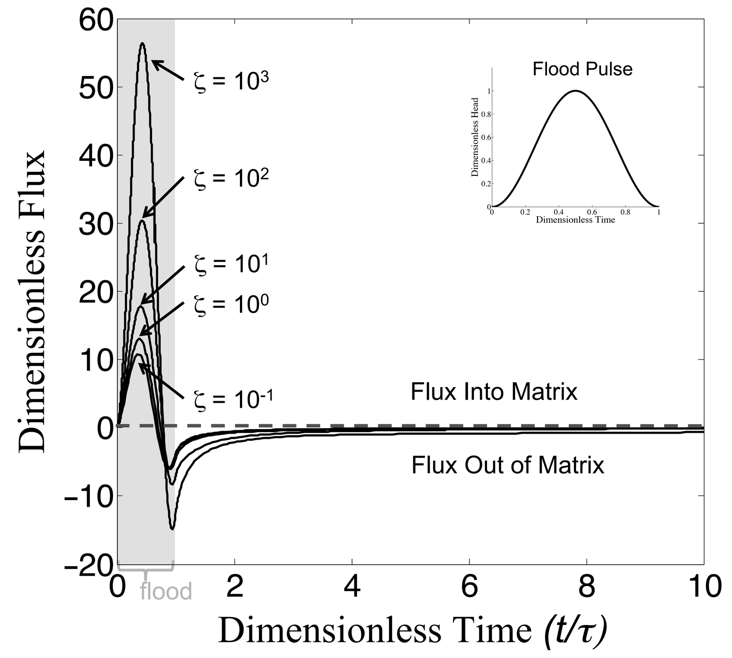

Figure 4: Flux of water to/from matrix during/after flood event. When the area under the curve above and below zero are equal the flood water volume will have drained from the matrix. Here ξ is a dimensionless parameter specifying flood, aquifer, and conduit properties. $\xi = (K\tau)/(S_s R^2)$, where: K= hydraulic conductivity; τ= flood pulse duration; S_s= specific storage; R= conduit radius.

Volume of Flood Water Stored

Flood water is stored in the matrix for much longer than the flood duration. The volume of flood water stored in the matrix rises during the flood and then as the peak passes it drains slowly. Depending on aquifer properties, it can take three orders of magnitude longer than the flood pulse duration for the aquifer to fully drain the flood water volume.

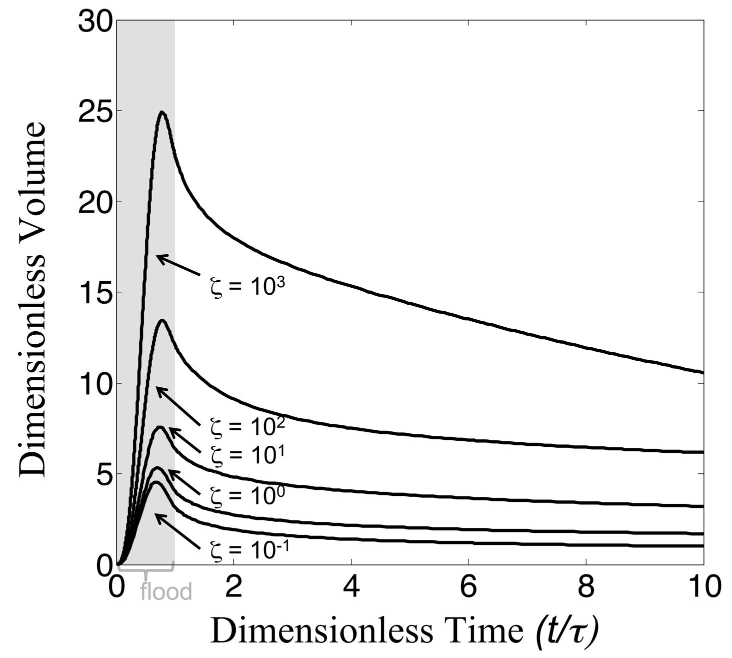

Figure 5: Volume of flood water in matrix due to conduit flood event. Flood water is very slow to drain from matrix after the flood event. Depending on aquifer properties, 15 - 45% of flood water volume remains in the aquifer at a time one order of magnitude longer than the flood pulse duration.

Head in Matrix

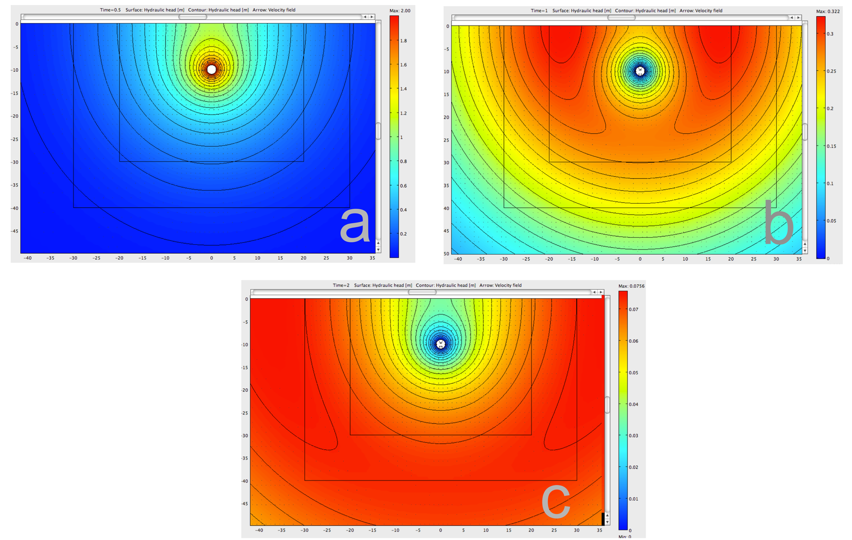

Figure 6: Head in matrix, red= high head. a) Half-way through flood pulse t/τ=0.5, (max. head = 2); b) at end of flood pulse, t/τ=1, (max. head = 0.3); c) after flood pulse, t/τ=2, (max. head = 0.08).

The head in the matrix generated by the flood is asymmetric and its distribution varies with time. At the end of the flood, $t = \tau$, the head close to the conduit, and far from the conduit is low, causing the region of high head to push water into the matrix and out to the conduit at the same time.

Mass of Flood Water Stored

Suppose the conduit flood water has a different chemistry than the matrix water. How much water in the matrix has the the chemical fingerprint of conduit water? Keep in mind that the volumetric storage of the flood wave involves a pressure or head pulse moving into (and back out of) the matrix, whereas storage of chemically tagged flood water involves displacement, dispersion, and diffusion. We test this using COMSOL's particle tracking algorithm and its finite element transport solution.

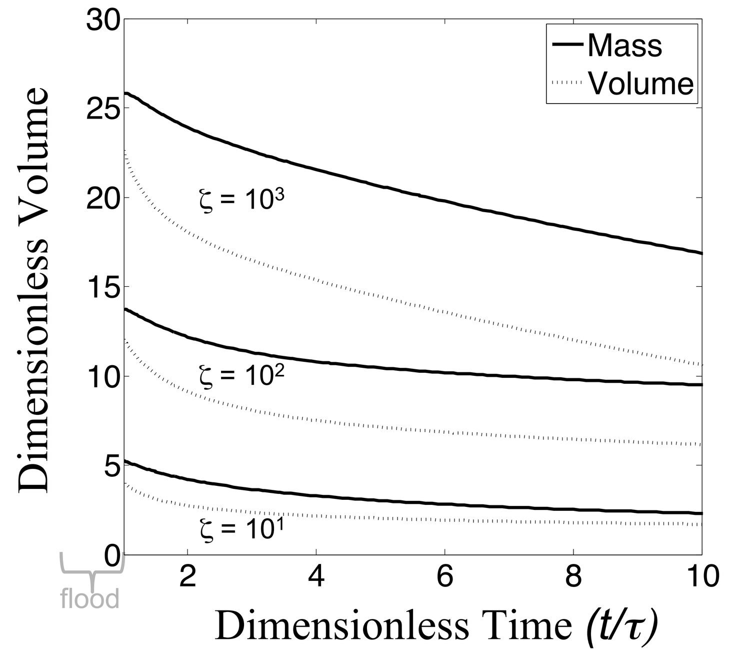

Figure 7: Flood mass and flood volume stored in matrix. Some of the water that drains from the matrix as a result of the flood is pre-flood matrix water; an equal volume of flood water remains in the matrix. Depending on aquifer properties, 5 - 20% of the flood water remains in the matrix at a time three orders of magnitude longer than the flood pulse.

Multiple Floods

Of more interest than a single flood event is what happens when there is a sequence of floods. Three flood pulses of different peak, duration, and inter-flood spacing were modeled, injecting a fresh set of particles along the conduit-matrix interface at the beginning of each new flood. The flood water does not drain completely from the matrix rock between floods. This leads to a build up of head in the matrix, as well as flood water mass. When the inter-arrival time of floods is less than three orders of magnitude longer than the flood duration, the subsequent flood pushes the flood water sequestered by the previous flood deeper into the matrix.

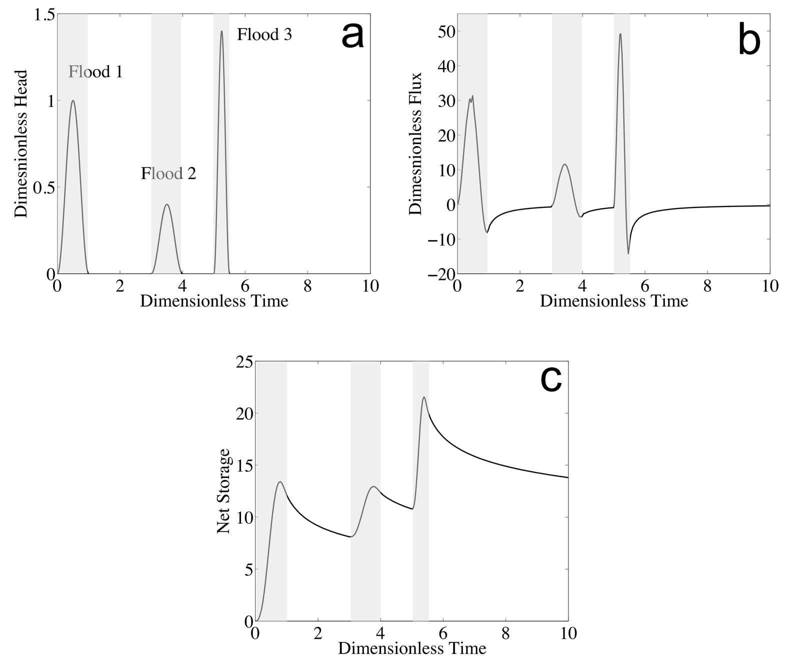

Figure 8: Multiple flood events with short to mid inter-arrival times. Subsequent floods push flood water sequestered by previous floods deeper into the matrix.

Objective & Questions

Objective: Model conduit-matrix exchange caused by conduit floods. A flood is a temporary increase of conduit pressure head.

Question 1: What are the fluxes between the conduit and matrix during and after a flood?

Question 2: What amount of flood water is stored in the matrix?

Question 3: What are the implications for contaminant sequestration?

Confined versus Unconfined Aquifer

The plots shown in Figures 4 - 8 are for the confined aquifer model. The unconfined aquifer model gives a different distribution of head in the matrix, Figure 9. This differing distribution of head leads to a faster flux of water into the matrix during the flood, and more flood volume stored. The after flood flux back to the conduit is still very slow.

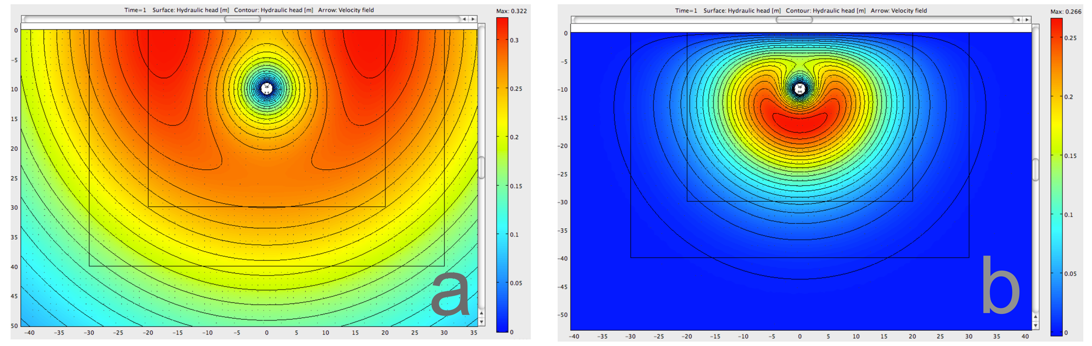

Figure 9: Head in matrix at end of flood pulse, t/τ=1, red= high head. a) confined aquifer (max. head =0.3); b) unconfined aquifer (max. head = 0.3).

Conclusions

1. Water is transferred to the matrix during the flood faster than it is transferred back to the conduit after the flood.
2. 20-50% (depending on aquifer properties) of the flood water remains in the matrix at a time one order of magnitude longer than the flood pulse duration. It can take over three orders of magnitude longer than the flood pulse duration for the aquifer to fully drain from the flood water volume.
3. Some of the water draining to the conduit due to the flood is pre-flood matrix water. This leads to a sequestration of up to 20% of the flood water in the matrix.

Future Work

- A sensitivity analysis of the model inlcuding the variables of conduit depth and radius.
- The model will be extended to 3-dimensions to include the dimension parallel to the conduit.
- Conduit behavior will be modeled, including the effects of conduit sinuosity and topography, and a vadose conduit case.
- Non-symmetric flood waves will be included.
- The model will be coupled with geochemical models to produce a speleogenetic model.
- Field data of conduit flood heads and geochemistry will be used to apply the results.

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